

General Certificate of Education  
June 2005  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 1**

**MPC1**

Tuesday 7 June 2005 Afternoon Session

**In addition to this paper you will require:**

- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

**Information**

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

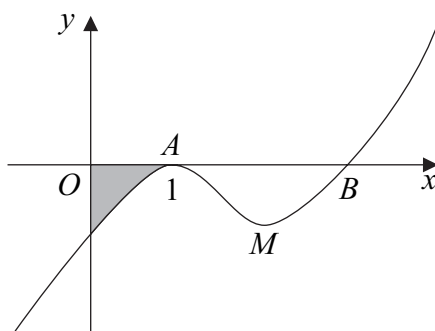
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Answer **all** questions.

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- 1** The point  $A$  has coordinates  $(6, 5)$  and the point  $B$  has coordinates  $(2, -1)$ .
- (a) Find the coordinates of the midpoint of  $AB$ . (2 marks)
- (b) Show that  $AB$  has length  $k\sqrt{13}$ , where  $k$  is an integer. (3 marks)
- (c) (i) Find the gradient of the line  $AB$ . (2 marks)
- (ii) Hence, or otherwise, show that the line  $AB$  has equation  $3x - 2y = 8$ . (2 marks)
- (d) The line  $AB$  intersects the line with equation  $2x + y = 10$  at the point  $C$ . Find the coordinates of  $C$ . (3 marks)
- 2** (a) Express  $x^2 - 6x + 16$  in the form  $(x - p)^2 + q$ . (2 marks)
- (b) A curve has equation  $y = x^2 - 6x + 16$ .
- Using your answer from part (a), or otherwise:
- (i) find the coordinates of the vertex (minimum point) of the curve; (2 marks)
- (ii) sketch the curve, indicating the value where the curve crosses the  $y$ -axis; (2 marks)
- (iii) state the equation of the line of symmetry of the curve. (1 mark)
- (c) Describe geometrically the transformation that maps the graph of  $y = x^2$  onto the graph of  $y = x^2 - 6x + 16$ . (3 marks)
- 3** A circle has centre  $C(2, -1)$  and radius 5. The point  $P$  has coordinates  $(6, 2)$ .
- (a) Write down an equation of the circle. (3 marks)
- (b) Verify that the point  $P$  lies on the circle. (2 marks)
- (c) Find the gradient of the line  $CP$ . (2 marks)
- (d) (i) Find the gradient of a line which is perpendicular to  $CP$ . (2 marks)
- (ii) Hence find an equation for the tangent to the circle at the point  $P$ . (1 mark)

4 The curve with equation  $y = x^3 - 5x^2 + 7x - 3$  is sketched below.



The curve touches the  $x$ -axis at the point  $A (1, 0)$  and cuts the  $x$ -axis at the point  $B$ .

(a) (i) Use the factor theorem to show that  $x - 3$  is a factor of

$$p(x) = x^3 - 5x^2 + 7x - 3 \quad (2 \text{ marks})$$

(ii) Hence find the coordinates of  $B$ . (1 mark)

(b) The point  $M$ , shown on the diagram, is a minimum point of the curve with equation  $y = x^3 - 5x^2 + 7x - 3$ .

(i) Find  $\frac{dy}{dx}$ . (2 marks)

(ii) Hence determine the  $x$ -coordinate of  $M$ . (3 marks)

(c) Find the value of  $\frac{d^2y}{dx^2}$  when  $x = 1$ . (2 marks)

(d) (i) Find  $\int (x^3 - 5x^2 + 7x - 3) dx$ . (4 marks)

(ii) Hence determine the area of the shaded region bounded by the curve and the coordinate axes. (4 marks)

5 Express each of the following in the form  $m + n\sqrt{3}$ , where  $m$  and  $n$  are integers:

(a)  $(\sqrt{3} + 1)^2$ ; (2 marks)

(b)  $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ . (3 marks)

- 6 The cubic polynomial  $p(x)$  is given by  $p(x) = (x - 2)(x^2 + x + 3)$ .
- (a) Show that  $p(x)$  can be written in the form  $x^3 + ax^2 + bx - 6$ , where  $a$  and  $b$  are constants whose values are to be found. (2 marks)
- (b) Use the Remainder Theorem to find the remainder when  $p(x)$  is divided by  $x + 1$ . (2 marks)
- (c) Prove that the equation  $(x - 2)(x^2 + x + 3) = 0$  has only one real root and state its value. (3 marks)

7 Solve each of the following inequalities:

- (a)  $3(x - 1) > 3 - 5(x + 6)$ ; (3 marks)
- (b)  $x^2 - x - 6 < 0$ . (4 marks)

8 A line has equation  $y = mx - 1$ , where  $m$  is a constant.

A curve has equation  $y = x^2 - 5x + 3$ .

- (a) Show that the  $x$ -coordinate of any point of intersection of the line and the curve satisfies the equation

$$x^2 - (5 + m)x + 4 = 0 \quad (1 \text{ mark})$$

- (b) Find the values of  $m$  for which the equation  $x^2 - (5 + m)x + 4 = 0$  has equal roots. (4 marks)
- (c) Describe geometrically the situation when  $m$  takes either of the values found in part (b). (1 mark)

**END OF QUESTIONS**